
Nicolas Drouhin, Vincent Touzé, Bruno Ventelou

KEY WORDS: AIDS; impact; macrodynamics; human capital; macroeconomic policy; endogenous growth model.

Abstract
The literature dealing with the issue of the macroeconomic impact of AIDS can be presented in several ways: by differentiating the studies according to the channels they use (section 1), or by differentiating the studies according to the development models that they use (section 2). We can then attempt to construct a “model” which synthesises these different approaches, thus enabling a comparative study of the means of forecasting the impact of AIDS in a developing economy (section 3). The result of this work is that, by considering a multiplicity of productivity variables as potential engines of development (such as human capital, public spending, etc.), endogenous growth models produce more valuable and precise assessments of an epidemiological crisis such as AIDS in developing countries. For example, we highlight a non-linearity and the risks of the persisting effect of the AIDS crisis on development, which would not be demonstrated by an exogenous growth model based solely on the idea of a “catching-up effect” established with regards to the permanent Gross Domestic Product (GDP) target level of the Solowian permanent regime.

Résumé
On peut présenter la littérature sur la question de l’impact macroéconomique du sida de plusieurs manières : en différenciant les études par les canaux qu’elles sollicitent (section 1), en différenciant les études par les modèles du
Introduction

AIDS is a terrible crisis, not only on a human level, but also on an economic level. Certain economists have understood this, for as early as the beginning of the 1990s they attempted to evaluate the economic impact of the epidemic: the impact appeared to be globally responsible for a reduction in the economic growth rate of one percentage point (cf. infra), with evaluations mainly considering the Gross Domestic Product (GDP) growth rate in countries affected by an epidemiological shock of HIV prevalence above 10%. The starting point for our own work is that these evaluations of growth reduction seem quite limited and that new evaluations are required: on the one hand, the relatively early date of the studies means that certain effects of AIDS on long-term economic behaviour are overlooked; on the other hand, the evaluation models used can be improved by systematically taking into account existing phenomena of productive complementarities (synergy) in the accumulation of productivity factors of the national economy (the different forms of “human capital” entering the production function, for example); these phenomena have notably been highlighted by the theory of endogenous growth and seem to have been underestimated in previous research on the economic impact of AIDS.

The literature on the macroeconomic impact of AIDS can be presented in several ways: by differentiating the studies according to the specific impact investigated (section 1), or according to the development models that they use (section 2). Finally, constructing a “model” which synthesises the different approaches will allow a comparative study of the means of forecasting the
impact of AIDS in developing economies and a more valid assessment of the impact of AIDS on these economies (section 3).

I
THE “IMPACT” APPROACH

At a macroeconomic level, the accumulation of wealth and the development of the economy could be negatively affected by a reduction in the average productivity of the country (reduction in life expectancy and alteration of the productivity of workers), but also by the increase in healthcare expenditure which “diverts” national resources from their long-term allocation. However, in the short run, healthcare expenditure is positively accounted in the GDP. To gain a clearer picture, we have applied a standard classification: direct cost / indirect cost, adding a supplementary category, deferred indirect cost [1].

Three impacts

Direct costs: These costs include medical expenditure linked to HIV/AIDS: treatment of opportunistic diseases as well as the possible use of antiretroviral drugs (ARVs), remuneration and training of medical and administrative personnel, spending on hospital infrastructure, monitoring and safety of blood transfusions, costs of preventive interventions, and costs of scientific and medical research (sometimes including the costs of prevention). In countries where there is some form of public health insurance and/or redistribution policy benefiting the poorest sections of the population, we can expect an increase in the tax rate to cover this increase in health expenditures. In Africa, these costs, said to be direct, will mainly be represented at a personal level (household income) by extra withdrawal from a ‘net surplus after consumption’ which is already excessively limited. The economic impact of AIDS will thus be felt mainly via savings, with the following mechanism:

AIDS → treatment → decrease in savings → lower accumulation of capital.

However, in many studies, this impact remains limited, as this “equation” is only true if AIDS incurs (costly) treatment. In the opposite case (illness not
treated or treated by traditional methods\(^1\), which is frequently the case in Africa), the economic effects of AIDS as a reduction in savings remain limited\(^2\).

**Indirect costs**: A person who develops AIDS experiences a reduction in his productive capacity, as he regularly falls ill. The reduction in his work capacity means a reduction in household resources if the victim has family responsibilities. For the poorest, this reduction is particularly hard to bear as opportunistic diseases increase the spending required on household healthcare (direct costs). In this case, the macroeconomic impact of AIDS is thus analysed through its impact on the labour supply, with the following mechanism:

\[
\text{AIDS} \rightarrow \text{invalidity} \rightarrow \text{reduction in labour participation.}
\]

In concrete terms, the consequences of AIDS on the productive system differ according to the production sectors: agriculture, transport, mining, manufacturing, the public sector. It appears indeed that analysis of the impact varies widely according to assumptions about the prevalence of HIV/AIDS in the different labour sectors, and about the possible susceptibility of the sectors in terms of labour flow, depending on their strategic importance for development. Some professional activities are more sensitive than others to epidemiological transmission of HIV, either for behavioural reasons (for example in the transport sector where employees, such as truck-drivers, are often a long way from home) or for reasons of exposure (for example, employees in the health sector). Another differentiating effect which causes concern is that in West Africa the prevalence of the disease is sometimes greater in skilled workers (for cultural reasons, also linked to the urban way of life\(^3\)). The public sector, with a concentration of skilled workers, could be particularly affected, especially the education

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1. Traditional methods are not cost free (symbolic sacrifice and travel expenses for example), but they cause minor *official* disbursements (visible in official statistics).

2. ...without counting the Malthusian effects: the demographic crisis (negative) can paradoxically play a favourable role for the economy measured by the per capita variable (of survivors); this argument recurs in the more precise study of the “impact” of AIDS in the Solow model. It is in this way that certain studies, such as that of Martha Ainsworth and Mead Over [2], insist on the “human cost” of AIDS, with numerous deaths, whilst admitting a relatively limited “economic cost”. See also the model developed by Cuddington and Hancock [3].

3. ...rather than rural for the less skilled employees (peasant workers). In the context of African economies, there are two important (linked) dualisms: urban / rural (agricultural), and skilled work / non-skilled work; a third, formal sector / informal sector could also be taken into account. See Cuddington and Hancock [3], Kambou, Devarajan and Over [4] and the studies conducted by the IIASA [5].
sector: according to UNESCO, AIDS is the main cause of mortality among teaching staff in Côte d'Ivoire. In general, the premature death of an employee leads to the loss of know-how which can no longer serve production or be transmitted. Therefore, even before a more detailed analysis of family strategies for access to education, there already appears in the analysis by sector the important question of the effects of AIDS on “human capital”, today considered as the boosting factor of economic and social development (see section 3 for an extensive use of “human capital” in a growth model [6]).

Deferred indirect costs: In the absence of external aid, the reduction in resources linked to AIDS could have even more significant deferred effects, as it risks modifying the productive structure of the household, notably its “educative” function. The spouse who is not ill (which would seem a highly unlikely situation) or ‘not yet ill’ will devote more time to work and less to the education of the children. The children themselves are sometimes diverted from their schooling and are forced to work; the “production detour” entailed in access to schooling would seem too costly in light of the financial emergency confronting the household. Deaths are also costly; the funeral rites can last for several days, necessitating de facto a prolonged interruption of work for the entire family. There remains the problem of the surviving children of the deceased. Who will provide for the needs of these orphans? Who will raise them and educate them? Generally speaking, a worrying characteristic of the AIDS crisis is its consequence on the accumulation-transmission of human capital throughout society: in other words, the quality of labour will not be assured in the long run.

All of these considerations, to which may be added an analysis of the age-pyramid of the population and its distortion due to AIDS (even more “hollowed out” in developing countries in the short run [7]), converge on the idea that AIDS affects:

1) in the short run, the quantity ratio: active-able to work / total population in the economy;

4. For the whole of Africa (not exclusively West Africa or Côte d’Ivoire), the debate on the AIDS-prevalence for skilled/unskilled workers is not clear. This question requires careful investigation by specialists. But it is not a crucial point for our own macro-dynamics approach, as in the long run the absence of creation-transmission of skill (deferred indirect costs) is more important than this short term destruction.
5. See below, the category “deferred indirect costs”.
6. Families often withdraw their daughters from school to take care of their sick parents or to fulfill other family responsibilities, thus compromising the education and future prospects of these young girls. In Swaziland, school enrolment has dropped by 36% due to AIDS, according to UNAIDS-WHO (cf. The facts on the AIDS epidemic, UNAIDS-WHO, December 2001).
In the long run, the quality of work supplied by the representative worker (the human capital effect).

Below, we will discuss the dual effect, quantitative (short-term) and qualitative (long-term), of AIDS on African economies.

**Some evaluations**

Different studies have been conducted to attempt to measure the economic consequences of HIV/AIDS in terms of GNP points lost. The main studies supply comparable figures for the African economies. On average, the authors forecast a one-point reduction in the growth rate of national wealth\(^7\). These studies are based on an *ad hoc* modelling of the economy, allowing a comparative approach: that is, a comparison between a “base-case scenario” (without AIDS) and the actual economy (with AIDS). The study conducted by Bonnel (2000) provides an econometric estimation which endeavours to link the growth rate to the prevalence rate by screening other explanatory factors such as the institutional environment, physical capital and human capital. This estimation of African economies used data collected between 1990 and 1997.

*Table 1: Link between growth, life expectancy and the prevalence rate*

<table>
<thead>
<tr>
<th>Rate of Prevalence</th>
<th>Reduction in the Rate of Per Capita Growth (1)</th>
<th>Years of Life Expectancy Lost (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>-0.6</td>
<td>4.7</td>
</tr>
<tr>
<td>10%</td>
<td>-0.8</td>
<td>9.4</td>
</tr>
<tr>
<td>15%</td>
<td>-1</td>
<td>14.1</td>
</tr>
<tr>
<td>20%</td>
<td>-1.2</td>
<td>18.8</td>
</tr>
<tr>
<td>30%</td>
<td>-1.4</td>
<td>28.2</td>
</tr>
</tbody>
</table>


7. The impact on the per capita GDP may appear less significant for the simple reason that AIDS reduces above all the size of the population, which represents a “positive” shock in the Malthusian context.
Table 2: Reduction in GNP attributable to HIV/AIDS

<table>
<thead>
<tr>
<th>Country</th>
<th>Average reduction in GNP (in annual growth points)</th>
<th>Period</th>
<th>Year</th>
<th>Sources/ authors</th>
</tr>
</thead>
</table>

Source: estimations collected by Touzé and Ventelou [1] using the cited articles; the intervals relate to the size of the impact according to the scenarios studied. A similar but more extensive table appears in Barnett and Whiteside [14], p 286-7.

In general (and despite a terrible human cost), these studies provide a “diagnosis” of a crisis which will remain on a relatively limited economic scale. First of all, it should be recalled that healthcare expenditure (like military expenditure) is included in the GDP: in the immediate context, there is a positive accounting effect, even if later it could be thought that this expenditure, re-directed towards emergency health-care, might hinder a balanced process of development. Next, it should be noted that more often than not these evaluations overlook “deferred indirect” effects (the transmission and accumulation of “capital”, both physical and human). In general, the problem of the aforementioned studies is that they do not offer a valid analysis in the long term: such analyses can only claim to be genuinely valid for a period of 5 to 10 years. This could be justified at the beginning of the 90s, as it was difficult to forecast exactly “the input” of the model over time, i.e. the epidemiological crisis. Furthermore, the studies

8. A notable exception is that of Barnett and Blaikie [15], who speak of AIDS as a “long wave” disaster on the same scale as global warming, insofar as the “major effects were already in play well before the extent of the crisis was known” and “no existing answer can be applied”.
are generally weak with regard to one of the first two channels cited earlier. In the long run, they neglect the possible interactions between the two (complementary) channels.

Indeed, the literature tends to neglect the analysis of the long-term effects of the crisis, and especially its impact on the accumulation of both physical and human capital. Thus, for these two variables, the impact exists and the risks are significant.

**Physical capital (direct cost):** the sums devoted to treatment are diverted from their productive allocations (savings and private investment; public investment). As we have seen, the effect is sometimes considered in the studies, but is often minimised due to a limited time horizon and to additional assumptions regarding Africa: low savings (constraint of indebtedness) and low productive investment.

**Human capital (indirect cost):** this effect is rarely evoked and in practice is not quantified in macroeconomic dynamics: on the one hand, as noted in the studies, AIDS reduces the productivity of the population, notably that of the skilled population, when there is a marked “dualism” for this type of work, while on the other hand it reduces the accumulation and transmission of skills (transmission via the family if the head of the family dies, via school if the teaching staff die; and the children, for their part, work at an earlier age due to the death of the head of the family).

And what if there were cumulative effects of these two variables? (i.e. the reciprocal relationship of human capital and physical capital during their formation process, and the need experienced by the poorest countries to develop these two engines of development concurrently?). We will return to this question in section 3, in which we give a brief explanation of the apparent paradox between the conventional forecasts of a limited aggregated impact, and the huge impact already measured at microeconomic levels (on businesses and households). For the moment, we can summarise our approach in the following way:

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9. Certain studies include a ‘skilled workers / unskilled workers’ differential [5], but few effects on the accumulation of skilled work. Only Theresa M. Ndongko [16] insists on the reduction of the expected future level of qualifications due to the fall in school attendance, but she does not measure it.
Table 3: Breakdown of the costs of HIV/AIDS

<table>
<thead>
<tr>
<th></th>
<th>Very poor countries</th>
<th>Developing countries</th>
<th>Rich countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct costs (1)</td>
<td>+</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>Indirect costs (2)</td>
<td>+++</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td>Deferred costs (3)</td>
<td>+++</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td>Prevention (4)</td>
<td>+</td>
<td>++</td>
<td>+++</td>
</tr>
</tbody>
</table>

(Hypothetical table*: + weak; ++ high; +++ very high)

1. These costs include medical expenditure linked to HIV/AIDS: treatment; remuneration of medical and administrative personnel; spending on hospital infrastructures; blood transfusion monitoring and safety; scientific research.

2. These costs include all immediate negative economic effects caused by the epidemic: reduction in productivity through the increase in morbidity, reduction in life expectancy (social and psychological damage).

3. These costs represent a profound loss of the development mechanisms due to a lack of accumulation of physical and human capital. They are noticeable in the long term.

4. These costs concern expenditure linked to informing the population about the disease and the practice of safe sex (use of condoms).

This table, both hypothetical and intuitive, summarises the idea of a differentiated impact of AIDS for the poorest countries. In contrast to rich countries, which are less affected and at the same time capable of a certain flexibility in their response to the disaster (“dualisms”, e.g. skilled work v. unskilled work, are less marked and the strategies for accumulating productive assets less constrained), the poorest countries will in all probability be confronted by extremely high deferred costs11. These remain invisible.

II

THE “STANDARD GROWTH MODEL” APPROACH

Before studying a more general model, it is important to present some relatively standard considerations regarding the long-term economic development analysis

10. See Touzé and Ventelou [1].

11. The main idea is that the development mechanisms of the poorest countries are fragile, and that a shock such as AIDS could have an enormous impact, not only on the current level of activity, but on the future development path of the country. Barnett and Whiteside [14] have used the term “susceptibility” to characterise this fragility.
models. Two major types of growth model can be distinguished: traditional models, with decreasing factorial returns, analysing growth as a process of catching up a target level determined by the scarcity of resources; and models with constant returns, also called “models of endogenous growth”, which seek in their analyses to constantly push back the resource scarcity frontier.

The impact of AIDS in an exogenous growth model: the Solowian framework

Traditional growth models study the relationship between the process of accumulation of capital and growth. They are characterised by a dynamic equation, called the Solow relation [17]:

\[ \dot{k}_t = s f(k_t, \pi) - n k_t \]

with \( k_t \) the capital per capita, \( \dot{k}_t \) its derivative in relation to time, \( s \) the rate of savings in the economy, \( \pi \) a term of productivity, \( f(k_t, \pi) \) per-capita production. This equation means that the net formation of capital per capita is equal to per-capita savings (\( sf(k_t, \pi) \)), less the share which serves to endow the new generations with capital (\( nk \)). The dynamics of the model result from the characteristics of the production function of the economy. What are these characteristics? Firstly, at a macroeconomic level, we generally assume that the production function presents constant returns to scale in its two arguments: capital and labour. This means that, potentially, the economy can reproduce itself identically from one period to the next, thus allowing extensive growth. Furthermore, as companies are constantly in search of efficient productive combinations and as these always contain capital and labour, this means that the production function is quasi-concave. This notably implies that the marginal productivity of factors (factorial returns) are decreasing and therefore that the production function per capita is a concave function of capital per capita.

In this framework, it is the relative scarcity of production factors which limits the growth process. As the only residual scarce factor is labour, it is precisely the dynamics of the population which will determine the dynamics of national income, possibly adjusted by the productivity growth rate (advances in knowledge). Capital per capita will evolve progressively to attain the target level in a steady-state regime, where the growth rate of both income and capital is equal to the growth rate of the population, again, as noted above, possibly increased by an exogenous productivity trend facilitated by
technological progress. Short-term growth is thus perceived as a process of catching up a target level determined by the scarce resources, whilst in the long term it is based on exogenous data and depends on neither the savings rate nor the growth rate of the population, and even less on other economic behaviour such as the choice of healthcare and education.

In such a context, what will the impact of the AIDS epidemic be on the dynamics of growth? It is necessary to consider the impact of AIDS on the parameters of the model. We will identify three effects, which can be represented in the equation and the following graph, with $\varepsilon$ representing the epidemiological crisis:

$$\dot{k}_i = s(\varepsilon) f\left(k_i, \pi(\varepsilon)\right) - n(\varepsilon)k_i$$

Figure 1: Impact of AIDS on growth: the traditional approach

- With regard to the population growth rate ($n$), we can logically assume that this will fall, even if we cannot exclude the fact that households modify their reproductive behaviour to compensate for the increased mortality. In the Solow model, this reduction in the growth rate of the population will have a counter-intuitive effect. Indeed, by reducing the demographic pressure on savings ($n k$), the fall in the population growth rate will tend to increase the

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12. We consider here an immediate impact. In practice, the modification of the parameters of the model will be progressive, according to the epidemiological model characterising the rate of prevalence.
capital per capita and will thus accelerate the growth of per-capita national income in transitional regime; this effect will then disappear in the steady-state regime. With regard to the growth of total income, the effect will be indeterminate in transitional regime (in the catch-up phase, $k \neq \bar{k}$), as the per capita income increases more quickly, while population growth is slower. In steady-state regime ($k = \bar{k}$), the growth of total income is weaker.

– With regard to the savings rate, two effects acting in the same direction can be noted. First, the increase in healthcare expenditure reduces the amount of resources available for savings and thus reduces per-capita savings. Second, according to the theory of the choice of savings in the life-cycle model with uncertain lifespan [18], an increase in the risk of dying reinforces the preference for the present and diminishes expected wealth, thus tending to reduce the rate of savings $^3$. In the Solow model, this reduction in the savings rate leads to a lower capital per capita product in the steady-state regime. It also reduces growth in the transitional regime. In the steady-state regime, it will have no effect on the growth of national income or of per-capita income.

– Finally, the AIDS epidemic certainly has a negative impact on labour productivity (absenteeism, fatigue, etc.), entailing a reduction in per capita production. This reduction leads to a fall in capital per capita for the permanent regime and thus a fall in growth in the transitory regime. However, in the permanent regime, the productivity growth rate remains exogenous and thus has no impact on the long-term growth of national income and national per-capita income.

In the final analysis, in the Solowian framework, the impact of the AIDS epidemic on capital per capita in a permanent regime is theoretically indeterminate since it is the sum of three effects, one positive and two negative. Only empirical quantification of these different effects can allow a conclusion to be drawn one way or the other. Furthermore, we note that the long-term growth rate of the economy is not affected, other than by demographic effects. We may nevertheless ask ourselves if studies analysing the impact of the AIDS epidemic on growth using the Solow model will not automatically lead to qualified

[^3]: The inter-temporal choice is also sensitive to the institutional context. For a detailed study of the comparative statics of the model of inter-temporal choice with an uncertain life span, see Drouhin [19].
conclusions due to the very structure of the model, notably as a result of the exogeneity of productivity growth. What happens if, for example, the productivity trend (human capital) and the savings rate (physical capital) both react in a cumulative manner, \( \pi \) and \( s \) being linked by microeconomic choices? The risk is seeing the development of negative synergies between the two variables in times of crisis, calling into question current assessments based on the hypothesis of separate effects.

The evolution of growth theory during the past twenty years has consisted of developing the Solow model to make long-term productivity development depend on the economic decisions of agents\(^1\). It is precisely the aim of the so-called endogenous growth theories to establish a new analytical framework giving a better description of agents behaviour, notably with regard to their choice of accumulation of factors, henceforth referred to as being of productivity, rather than of production (those already identified). In particular, the introduction of knowledge as an economic good or of knowledge embedded in the economic agents (human capital) is likely to counteract the effect of the relative scarcity of the labour factor on growth\(^2\). The growth rate of the economy will thus be determined endogenously by the decisions of interacting economic agents. In such a context, the AIDS epidemic can have much more marked consequences in terms of growth, as it will interact with decisions about healthcare and the accumulation of human capital, and thus on the growth rate of the economy.

The comparative statics which we have just evoked in the Solow model (and which assumed a study of the impact on economic variables decided individually: \( \pi \), \( s \) and \( n \)) can thus be improved by considering:

- genuinely endogenous dynamics of labour productivity;
- the possibility that, in addition to the effect of changes in labour productivity, the other key variables of development (savings rate, demographic structure, etc.) interact – not only additively – in the face of the crisis, and do so via the microeconomic choices of the decentralised agents.

By increasing the variables of productivity as potential engines of development, the endogenous growth models produce assessments of epidemiological crises, such as AIDS in developing countries, which are potentially both more useful and more precise. For example, in the following section, we will see the appearance

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\(^1\) And thus also on the institutional context where they are taken.

\(^2\) In other words, as certain factors which can be taken together appear with a non-decreasing return in the macroeconomic production function, there are no longer any scarce factors [20].
of non-linearity and the risk of the persisting effect of the AIDS crisis on development, a fact that an exogenous growth model could not take into account because it remains based on the idea of “catching-up” in relation to the target level of a single permanent regime. In an endogenous model, there are several permanent regimes, and a possibility of divergence rather than convergence.

**The impact of demography in an AK-type endogenous growth model**

In an endogenous growth model, the productivity of factors of production, notably that of labour ($\pi_t$), is linked to the evolution of the different variables of the economy. In its most simple formulation, labour productivity is assumed to have a linear relationship with the stock of available capital: this relationship can be expressed as $\pi_t = \pi_t \cdot K_t$, where $\pi_t$ is a residual and exogenous parameter of productivity growth. Using a Cobb-Douglas production function, national production also becomes a linear function of the stock of capital:

$$Y_t = F(K_t, \pi_t, L_t) = (L_t \pi_t)^{1-a} (K_t).$$

This formulation brings us back to the model called “AK”, $Y_t = A_t \cdot K_t$, where $A_t = (L_t \pi_t)^{1-a}$. The technique employed here is not that of constant returns to scale (Solow model) but of crossing, and the marginal productivity of capital is no longer a decreasing function of the stock of capital. Indeed, the accumulation of capital indirectly (via its effects on $\pi$) produces gains in global productivity of factors which are sufficient to compensate for the decrease in the marginal productivity of capital. However, the marginal productivity of labour remains a decreasing function of the number of workers.

The dynamics of capital accumulation is described by the following equation:

$$K_{t+1} = K_t + s_t \cdot (L_t \pi_t)^{1-a} \cdot K_t.$$  

where $s_t$ is the savings rate. The growth rate of the capital stock is calculated as follows:

$$\frac{\Delta K_{t+1}}{K_t} = s_t \cdot (\pi_t \cdot L_t)^{1-a}.$$
This equation demonstrates that the size of the active population influences the growth rate of the capital stock: this is “productive synergy” which can be explained by the fact that the gains in global productivity of the factors caused by the accumulation of capital become greater as the size of the active population increases: “the positive externality” is shared by a larger number of workers which reduces its impact. The parameter \((1-\alpha)\) measures the elasticity of the growth rate to population size.

The growth rate of per-capita production can also be calculated:

\[
\frac{\Delta Y_t / L_t}{Y_{t-1} / L_{t-1}} = \frac{\Delta A_t}{A_{t-1}} + \frac{\Delta K_t}{K_{t-1}} - \frac{\Delta L_t}{L_{t-1}}.
\]

Insofar as the growth rate of the global productivity of capital can be approached in the following way: \(\frac{\Delta A_t}{A_{t-1}}= (1-\alpha)\left(\frac{\Delta \pi_t}{\pi_{t-1}} + \frac{\Delta L_t}{L_{t-1}}\right)\), it is possible to deduce the growth rate of per capita income as the sum of three effects:

\[
\frac{\Delta Y_t / L_t}{Y_{t-1} / L_{t-1}} = s\cdot \left(\frac{\pi_t}{L_t} \right)^{1-\alpha} + (1-\alpha)\left(\frac{\Delta \pi_t}{\pi_{t-1}} - \alpha \cdot \frac{\Delta L_t}{L_{t-1}}\right).
\]

The first term refers to the growth rate of the capital stock. The other two effects form a weighted average between the increase in the exogenous component of productivity (favourable to the growth of per-capita income) and the relative variation of the active population (unfavourable Malthusian effect).

Following a single variation of the active population, we encounter the Malthusian effect characteristic of the Solow model. The latter appears as a transitional crisis in the form \(-\alpha \cdot \frac{\Delta L_t}{L_{t-1}}\): the decrease in the active population initially indicates an increase in the growth of production per capita. Nevertheless, for reasons of productive synergy (a hypothesis contained in the AK model), a larger population is favourable to a high growth rate of per capita income. Thus, with a capital elasticity coefficient equal to 0.3, a permanent reduction of 10% in the size of the population represents a transitional 3-point increase in the growth rate of per-capita income followed by a permanent reduction of 7% in the level of the growth rate (cf. elasticity of the growth rate.
of the capital stock to population size). For example, if the level of the long-term endogenous growth rate is 5%, we observe a temporary increase (Malthusian effect) of +3 points (at the moment of the crisis, the growth rate reaches 8%) accompanied by a permanent reduction of 0.35 points (from the moment of the crisis to the “end of time” the growth rate is reduced to 4.65%). In the space of 10 periods, the “synergy” effect becomes preponderant and the level of production is thus definitively lower in relation to its initial course before the crisis.

III
A COMPREHENSIVE MODEL.

Discussion on the choice of a model

Faced with those multiplicity of effects, our own work firstly seeks to synthesise the different approaches, with the initial objective of better evaluating the relative importance of each of the impacts, and also with a view to obtaining and quantifying their possible resonance effects when the accumulation profiles of several factors overlap.

The work thus aims to take into consideration both the different channels of impacts and the different models. To summarise its structure, the best solution is to list the effects (the “channels”) and to indicate which variable will be affected first by the crisis.

<table>
<thead>
<tr>
<th>CHANNEL</th>
<th>VARIABLE OF THE MODEL</th>
<th>TIME HORIZON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of participation (quantitative effect of AIDS on the labour supply)</td>
<td>( L/N ), share of the total labour force effectively able of working</td>
<td>Short</td>
</tr>
<tr>
<td>Productivity of workers (qualitative effects of AIDS on the labour supply)</td>
<td>( H ), “human capital”</td>
<td>Long</td>
</tr>
<tr>
<td>Rate of public investment</td>
<td>( D ), productive public spending</td>
<td>Long</td>
</tr>
<tr>
<td>Private investment</td>
<td>( K ), physical capital accumulated in the short term (possible financial imbalance of the national economy)</td>
<td>Short</td>
</tr>
<tr>
<td>Rate of private savings</td>
<td>( K ), physical capital available in the long term (economy financially balanced: endogenous interest rate)</td>
<td>Long</td>
</tr>
</tbody>
</table>
With the following macroeconomic production function:

\[ Y = F(K, L, D, H) = K^\alpha (L \cdot H)^\beta (D)^\gamma \]

To present a phenomenon of endogenous growth, the production function should be characterised by the constraint: \( \alpha + \beta + \gamma = 1 \). We should note that it is still possible to observe and compare this type of model with the forecasts of an exogenous growth model, by assuming the alternative hypothesis: \( \alpha + \beta + \gamma < 1 \).

In the appendix, we present the microeconomic specificities of the model: choice for \( H \), choice for \( K \), choice for \( D \). These variables are “endogenised” by the microeconomic choices and are thus possibly linked to each other (to simplify, we note that each variable depends on income \( Y \)). We also note \( \varepsilon \), the epidemiological state of the population. Any increase in \( \varepsilon \) represents a deterioration in the health status of the population.

\[ Y(\varepsilon) = (K(Y, \varepsilon))^\alpha (L(Y, \varepsilon) \cdot H(Y, \varepsilon))^\beta (D(Y, \varepsilon))^{1-\alpha-\beta} \]

This equation will serve to analyse the effect of the crisis on the macroeconomic income of a country. We should note that, in comparison with the Solow model, a separate analysis of the different impacts (per isolated variable) is not possible directly, as the cumulative effects passing via \( Y \) and the endogenous loop would not be taken into account. It is necessary to resolve the model first.

**A result: epidemiological trap and threshold effect**

Leaving aside the numerical and calculable applications of the model (we confine ourselves here to an analytical resolution), we can first simplify the general model presented in the appendix in order to explain as clearly as possible what is referred to as the risk of an “epidemiological trap”. For the case which concerns us (demonstrating a trap), we can, without losing any generality, reduce the model to two factors of production:

16. Here, we will analyse the increase in \( \varepsilon \) as the “AIDS crisis”. However, the same reasoning can be applied to any pandemic which affects both the virtual prognosis and the capacity to work: tuberculosis, malaria, yellow fever, etc.
$Y = K^\alpha \left( h(\varepsilon, x^-) L \right)^{1-\alpha}$

- With \( h \) the level of individual human (health) capital of workers, we obtain \( h'_{\varepsilon} < 0 \) (AIDS destroys human capital) and \( h'_{x^-} > 0 \) (human capital can be constituted and reconstituted thanks to healthcare expenditure \( x \)).

- We also assume that:

$$L = \left[ 1 - \varepsilon \left( 1 - g(\varepsilon^-) \right) \right] N$$

The number of agents \( L \) effectively able to work in an economy with \( N \) potential workers is affected by AIDS: the factor \( \varepsilon \) has a negative effect on the ratio \( L/N \) (active/total population). However, we can possibly restore the labour capacity by using healthcare \( x \): the logarithmic function \( g \) provides the size of this restoration. We assume \( g' > 0, g'' < 0 \), with \( g(0) = 0 \) and \( g(\infty) = g^* < 1 \).

Two “health effects” are present: a qualitative effect (AIDS affects the productivity of workers via human capital, \( h \)) and a quantitative effect (AIDS has a short-term effect on the ratio active/total population, \( \varepsilon (1-g) \)).

We establish a very simple expression of health expenditure:

$$x = \lambda \cdot y$$

and a human capital formation function which is just as simple:

$$h = x^- \cdot l(\varepsilon^-)$$

with \( l(\varepsilon) \) a measure of the impact of AIDS on the state of individual health: \( l' < 0 \).

These two functions can be justified by more complex ‘microeconomics’, but the important element is the strong direction of the variation of the functions (see appendix for more details).

Based on the assumption of the profit maximisation of firms, the productive capital stock can be written as a proportion of national income:
We can thus very simply derive the growth rate of national per-capita income:

\[ 1 + G = \frac{y}{y^*} = \Gamma \left( e^{-}, e, \mu, y^{-} \right) \]

where:

\[ \Gamma (e^{-}, e, \mu, y^{-}) = \lambda \left( e^{-} \left( 1 - e \left( 1 - g \left( \lambda y^{-} \right) \right) \right) \right) \left( \frac{\alpha (1 - \mu)}{r} \right)^{\alpha - 1} \]

The growth rate of the economy \( G \) is therefore an increasing function of the already attained per-capita national income (\( y^* \)), varying between a maximum value \( \bar{G} \) and a minimum value \( \underline{G} \).

Here, we wish to stress a particularly interesting case which is likely to modify noticeably the manner in which we envisage the impact of AIDS on the growth of the economy: when \( \underline{G} \) and \( \bar{G} \) are situated on either side of the value one, we notice the appearance of a threshold effect\(^{17}\).

\[ Figure 2: Growth and the threshold effect \]

\[ 14G \]

\[ 1 + G \]

\[ 1 \]

\[ 1 + G \]

\[ y \]

\[ y^* \]

\(^{17}\) We should stress that this case corresponds to realistic values of the parameters. Numeric measurement of the complete model presented in the appendix is currently being carried out for Côte d’Ivoire.
For an epidemiological state of the population, $\varepsilon$, and for a given level of health expenditure $\lambda$, a threshold $y_c$ exists, below which the growth rate of per-capita national income is negative. The “withdrawal” caused by the disease and a poor level of health on the factors of productivity of the economy are too serious to allow the latter to reproduce identically. However, above this threshold, the economy provides sufficient resources to undertake a process of self-maintained cumulative growth (benefiting from the regular provision of human capital parallel to the accumulation of physical capital). In this model, an under-development trap appears which we may qualify as an epidemiological trap.

What are the effects of the AIDS epidemic in this analysis framework? The increase in $\varepsilon$ shifts the growth rate curve downwards, mechanically raising the critical threshold for per capita national income below which the growth process is inversed. For countries initially experiencing growth, there are two possible scenarios.

In the first scenario, which we qualify as “depressive”, the threshold $y_c$, although raised, remains below the current level of per capita national income at the moment of the crisis. Here, the growth rate of the economy will be lower for all future periods. In particular it will converge on a permanent regime rate below that which would have been possible in the absence of the epidemic.

Figure 3: The depressive scenario
This scenario, which concerns the countries which are more advanced in their development process and/or where the epidemiological impact remains limited, provides a vision of the impact of the AIDS epidemic which, although significant, remains a case of a differential in the growth rate. Growth slows down, but the process of growth itself is not fundamentally altered in its essential mechanisms.

The second scenario, which we qualify as “catastrophic” in the etymological sense of the word, is quite different. In this scenario, which concerns countries which are still at the very beginning of their development process and/or where the AIDS epidemic has reached significant proportions, the AIDS crisis will increase the critical threshold $y_C$ above the per-capita national income level existing at the moment of the epidemiological crisis. A bifurcation is thus produced. The process of growth stops and the economy begins to regress. At a basic level, economic agents modify their long-term prospects (stationary equilibrium has changed) and the entire economy heads towards a reverse low equilibrium (characterised by a growth rate level of $1 + G$).

Figure 4: The catastrophic scenario
Our model strongly emphasises that not all countries are affected by the epidemic in the same way. If the AIDS crisis radically modifies the dynamics of the economy, it would be a serious mistake to take an approach which limits itself to measuring the distances between the growth rate curves for the “base-case scenario” (before the AIDS crisis) and the growth rate curve after the crisis, as it would overlook the bifurcation. In other words, the comparison would be made with a base-case scenario (“no-AIDS”) which already (incorrectly) includes the impact of the crisis! In the catastrophic scenario, the question is no longer how long it will take to reach a given level of development. What is at stake is quite simply the potential loss of any opportunity for further economic development in the countries concerned (in the absence, of course, of positive counter-shocks, such as massive technological transfers, notably in the medical domain; see below, ARV programmes).

Taking into account this possibility noticeably modifies the analyses of health policies which can be implemented. The coefficient $\lambda$, i.e. the proportion of per capita national income devoted to healthcare expenditure is the variable which allows the partial compensation of the impact of the AIDS epidemic. The increase in healthcare spending acts not only on the rate of participation (quantitative effect) but also on productivity through human capital (qualitative effect). It must nevertheless be stressed that no level of healthcare spending exists which would totally cancel the effect of the disease on growth, for two reasons. The first results from medical technology which, in the current state of knowledge, does not allow the consequences of the disease to be “repaired in their entirety”. This first reason is presented in our model by means of the function $g(\lambda y^-)$ which is bounded at a level $g^*$ which is strictly below 1. The second reason, which is more fundamentally economic, results from the necessity, in the absence of external aid, to finance this supplementary healthcare spending. In our simplified model, where there is no other public spending, we can simply consider that the tax levied at a rate of $\mu$ serves to finance healthcare spending: $\mu = \lambda$. If we adopt the maximisation of the growth rate of the economy as a criterion of well-being, we will have an optimum level of healthcare spending $\hat{\lambda}$, taking into account the opportunity cost of public funds. The effect of healthcare spending can thus be illustrated graphically. For all values $\lambda < \hat{\lambda}$, the increase in healthcare spending allows the growth rate curve to be “lifted”, thus softening the economic effect of the AIDS crisis. However, for higher values of $\lambda$, the effect of the tax deduction for healthcare spending on the accumulation of factors of
productivity becomes too significant and does more than cancel the beneficial
effects of the latter. Graphically, this increase in $\lambda$ is represented by a fall in
the growth rate curve. It should be added that any increase which might, as an
ethical argument, tend dogmatically to refuse the existence of this optimum $\lambda$
would commit a grave error of reasoning. The growth of the economy is the
only sustainable means of increasing healthcare spending. In this domain, as
in many others, the choice of economic policy is also an inter-temporal trade-
off. However, it is quite obvious to us that the effective $\lambda$ practised by develop-
ing countries is still well below the threshold level ($\lambda < \hat{\lambda}$) and that a pro-
gression margin is obviously desirable for healthcare spending in order to lift
the growth curve and remove the risks of an epidemiological trap.

Furthermore, we wish to stress that our model also supplies an argument
justifying the increase of healthcare spending above $\hat{\lambda}$, provided that this
spending is financed by international transfers. Indeed, if we consider the
catastrophic scenario and we choose healthcare spending which maximises
the growth rate, taking into account the AIDS crisis, there is nothing to say
that this maximum rate might not, despite everything, be negative! In this
situation, since any increase in healthcare spending financed by internal resources
would reduce the growth rate, only international transfers financing the over-
spending on health would allow the bifurcation of the economy to be avoided
and the growth process to be maintained. Of course, this over-spending would
only be transitional, as once the per capita GDP is pushed above the critical
threshold, the country would be in a position to finance its health policy itself.

By putting forward the possibility of an epidemiological trap, we provide
arguments which highlight the particularity of the effect of the AIDS epidemic on
the least developed countries (or on the most affected low – and medium –
income countries) in their development process; a particularity which requires
public policy measures which are themselves particular. To be precise, our ar-
gument, essentially based on a macro-dynamic analysis, offers strong support
for therapeutic ARV programmes: the outcome of such programmes could be, as
an emergency policy, to remove the epidemiological trap (and, as a positive
spin-off, to re-validate the linear prognostics delivered by the classic models of
forecasting!).
Conclusion

The simplified model in section III was based on two crucial assumptions:

i) AIDS has a short-term impact on a flow variable (the flow of potential workers available and able to effectively work at a specific time $t$ in the economy);

ii) AIDS has a long-term impact on a stock variable (human capital, i.e. the stock of the workers’ state of health or education). Integrating these two impacts in a coherent production model, is sufficient to reverse the standard prognostic based on a relatively linear evolution of the economies. An involution trap appears, corresponding to a modification of the long-term growth regime of the economy (the equilibrium which we aim for):

Figure 5: Economic Impact of AIDS: two paths for the economy

AIDS

(Intensity of the crisis and/or weakness of the health policy response)

Above the epidemiological threshold:
Growth and development

- more production
- more productivity
- more workers
- more spending in (human) capital

Below the epidemiological threshold:
Trap and involution

- less production
- less productivity
- fewer workers
- less spending in (human) capital

The quantifications of the macroeconomic impact of AIDS on developing countries are only in the early stages. The sensitivity of the studies to the structural models used for forecasting is a delicate point. Notably the appearance of threshold effects and bifurcations calls into question the prognostics based on economies which are developing much more regularly. Thus, by using forecasting terminology, it would appear that the base-case scenario, i.e. the
virtual evolution of the economy for a supposed zero crisis, is totally disrupted, after a bifurcation, by the crisis itself. This phenomenon invalidates the impact quantifications based on comparisons by linear approximation. Economic behaviour, incorporating the shift of the long-term equilibrium due to the crisis (the bifurcation), is already altered\(^\text{18}\), making the use of the base-case scenario futile, or at the very least misleading.

**APPENDIX**

**COMPLETE PRESENTATION OF THE MODEL**

*Production function and behaviour of firms*

The production function describes a technology with constant returns to scale and includes four arguments: the stock of physical capital \(K\), the number of workers \(L\), productive public spending \(D\) and the stock of human capital \(H\). It is expressed as:

\[
Y = F(K, L, D, H) = K^\alpha (L.H)^\beta (D)^{1-\alpha-\beta}
\]

We assume that firms maximise their profit. In the optimum situation, the private factors of production are remunerated at their marginal productivities. This gives:

\[
(1 - \mu) \frac{\partial Y}{\partial K} = r \Rightarrow (1 - \mu)\alpha \frac{Y}{K} = r
\]

\[
(1 - \mu) \frac{\partial Y}{\partial L} = w \Rightarrow (1 - \mu)\beta \frac{Y}{L} = w
\]

Notes: \(H\) and \(D\) behaviour will be determined respectively by households and the government. \(L\) is given by a population dynamic which is altered by AIDS, for a total population of \(N\) individuals. These demographic dynamics are described by:

\[
\frac{L}{N} = 1 - e(1 - g(x^-));
\]

\(^\text{18}\) This argument is similar to that evoked in the “Lucas criticism” [21] of econometric models.
with $\varepsilon$ AIDS crisis, and $g(x)$ a “reparation function” of the ability of AIDS sufferers to work through healthcare spending ($x$ in t-1). We stipulate the following restrictions:

$$g'_x > 0 \text{ et } 0 < g(x) < 1.$$

**Programme and behaviour of the different types of household**

We consider two types of household: the workers-employees and the capitalists-rentiers.

**Behaviour of workers-employees**

Employees do not save. We consider the plan of an extended family, of which a proportion $L/N$ works (the different families are affected identically by the disease). There are $F$ families and $N/F$ is the number of individuals in the household.

Decisions about household consumption rest on a choice between consumption of everyday goods and consumption of health goods. Health demands are influenced by the morbidity of the family. The function $\ell(\varepsilon)$ influences the state of family health: we have $e = x\ell(\varepsilon)$ with the restrictions: $\ell'_e < 0$, et $\ell(1) = 0$. The programme of maximisation of a household is expressed:

$$
\begin{align*}
\text{Max} [U(c,e)] &= \text{Max} [(v_c \varepsilon^\theta + \rho \varepsilon^\phi)^{\frac{L}{F}}] \\
sc : (N/F)c + p(N/F)x &= w(L/F) + T/F
\end{align*}
$$

We use $w$ as the wage rate, $T$ transfers, $p$ the price of health/education goods, $x$ the private quantities of health/education goods consumed, $e$ the state of health/education.

The stock of human capital evolves following the rule: $H^+ = e + \delta H$. $e$ is a flow of consumption of health and/or educational goods, which accumulates on the stock of human capital $H$. This stock $H$ will itself be used as an individual indicator of the quality of labour. This effect can “capture” both a health-capital effect and the restoring of this capital-health by healthcare spending, and an education effect.

We find:

$$
\frac{vc^{\phi-1}}{\rho \varepsilon^{\phi-1} \ell(\varepsilon)} = \frac{1}{p}
$$

$$
\begin{align*}
c + px &= w(L/N) + T/N
\end{align*}
$$
and the function of health status demands is written:

\[ e = \frac{w(L/N) + T/N}{\frac{p}{l(e)} + \left(\frac{\nu}{\rho}\right)^{1-\phi} \left(\frac{p}{l(e)}\right)^{1-\phi}} \]

Behaviour of the capitalists-rentiers (endogenous saving)

The capitalists have a consumption/savings choice. They seek to maximise their inter-temporal welfare. The programme is written:

\[ W = \text{Max}[U(c) + \frac{1-\varepsilon}{1+R} W^+] \]

\[ sc : c = rk - (k^+ - k) + rb - (b^+ - b) + \pi \]

where \( r \) is the rate of return on the world capital market (assumption of a small open economy), \( k \) is the accumulation of national assets, \( b \) foreign asset holdings. The programme gives in log form (for example) the following profile of inter-temporal consumption:

\[ \frac{c^+}{c} = \frac{1-\varepsilon}{1+R} (1+r) \]

“Behaviour” of the State

The functions of the State are limited to financing public expenditure \((D^+)\) with a tax rate \((\mu)\) on added value \((Y)\), and which is only productive for the period that follows, as well as subsidising healthcare spending on a flat-rate basis \((T)\), but this spending can also be assimilated to the financing of a collective health infrastructure:

\[ \mu Y = D^+ + T \]
Major lines of resolution of the model

Based on the production function, \( Y = (K)^{\alpha} (LH)^{\beta} (D)^{1-\alpha-\beta} \), we can substitute the stock of capital by \( K = \frac{\alpha(1-\mu)}{r} Y \), the number of workers by \( L = (1-\varepsilon(1-g(x^-)))N \), the stock of human capital by \( H^+ = e + \delta H \) and productive public spending by \( D^+ = \mu Y - T \).

We thus find:
\[
Y = \left( \frac{\alpha(1-\mu)}{r} \right)^{\alpha/1-\alpha} \left[ N(1-\varepsilon(1-g(\frac{e^-}{l(e^-)}))(e^- + \delta H^-))^{\beta/1-\alpha} (\mu Y^- - T^-) \right]^{(1-\alpha-\beta)/1-\alpha}
\]

We note that \( Y \) is a function of delayed variables (\( Y^- \) and \( \varepsilon^- \)). We could thus very simply calculate the GDP of the period (and the GDP growth rate) with the help of numeric simulations.

By recalling the demand function of the state of health calculated below:
\[
e^- = \frac{w^- (L^- / N^-) + T^- / N^- \frac{\rho^\frac{1}{1-\psi}}{l(e^-)^{\frac{1}{1-\psi}}}}{\frac{p^-}{l(e^-)} + (\frac{\rho}{p^-})^{\frac{1}{1-\psi}}}
\]
we highlight some interesting results to be commented upon. With regard to the question of public choice (\( T \) for example), we note the trade-off on \( D \) versus \( H \) (accumulation of \( e \)), fundamentally regulated by the production elasticity of the two factors.

Note 1: a priori, an under-estimation by the public decision-maker of the coefficient \( \beta \) (elasticity of production to human capital) biases the public choice to the detriment of \( H \) and health/education spending.

Note 2: a perverse effect: when \( \varepsilon \) increases, the number of workers falls and the public choice on \( H \) could also be biased downwards.
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